# Cluster Information based User Scheduling for Multiuser MIMO Systems

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Abstract-Two efficient user scheduling metrics and their corresponding algorithms are proposed for uplink multiuser MIMO systems in a finite scattering environment, where users typically transmit signals while sharing some clusters (which can be trees and buildings). By utilizing statistical channel state information such as how users employ clusters (e.g., percentage of energy carried by each cluster) and spatial correlation matrices of channels via each cluster, two scheduling metrics which achieve a tradeoff between decreasing intra-link correlation and increasing inter-link correlation are proposed. Two algorithms based on the proposed metrics are further designed. The proposed algorithms are of low complexity and can be executed in a low frequency through utilizing simple operations and statistical channel information, which brings in a great computational complexity reduction. Simulation results show that both algorithms could achieve significant sum rate gains comparing with other existing algorithms, and the performance can be close to that of the optimal scheme.

## I. INTRODUCTION

Multi-user multiple input and multiple output (MU-MIMO) systems have been drawing a lot of attention because of its high spectral efficiency and diversity gain [1][2]. A base station (BS) equipped with multiple antennas can communicate with multiple users in the same time-frequency slot via spacedivision multiple access, providing a substantial gain to system throughput [3]. However, the number of users which can be simultaneously served is limited by BS antenna number, due to the utilizing of block-diagonalization zero-forcing (BD-ZF) schemes which can eliminate inter-user interference [4]. Therefore, it is necessary to design user scheduling scheme to select the optimal user set in order to maximize system throughput in MU-MIMO systems.

The optimal user set can be obtained by a brute-force traversal search, with a prohibitive computational burden. In order to reduce complexity, several suboptimal strategies have been proposed [5–7]. Capacity-based algorithm in [5] selects users iteratively to maximize system capacity, but the complexity is too high since a large number of singular value decomposition (SVD) operations are required. Frobenius norm based algorithm is further proposed so as to reduce complexity, however, the performance degradation is inevitable. Orthogonality is also an important factor in the scheduling metric design. The authors in [6] propose a scheduling method based on chordal distance (a metric which measures orthogonality among users' signal subspaces), and it achieves a performance

close to that of capacity-based algorithm. In [7], an algorithm is proposed to select users according to the orthogonality between principle eigenvectors of users' channel matrices, and it achieves a capacity close to that of the optimal strategy. However, all above orthogonality measurements require a great many SVD operations, which compromises their practicality. Furthermore, these algorithms in [5–7] have to update their selected user subset as instantaneous channel state information (CSI) changes, which causes a high update frequency and thus a heavy computational burden.

In this paper, we investigate the problem of statistical CSI based scheduling for uplink MU-MIMO systems in a finite scattering environment. In real propagation environment, scatterers can be leaves, walls and cars, etc., and several scatterers close to each other can be treated as a cluster, such as trees and a building. For a multiuser system in a finite scattering environment, many users may have to share some common clusters to transmit signals. The main contributions of this paper are summarized as follow.

- Two statistical CSI based metrics that achieve a tradeoff between decreasing intra-link correlation and increasing inter-link correlation are proposed. They utilize statistical CSI like percentage of energy carried by each cluster, and correlation matrix of channel via each cluster. The metrics are of low computational complexity by avoiding SVD operations.
- Based on the two metrics, two corresponding scheduling algorithms that select users iteratively are designed. Simulation results indicate that the proposed algorithms could achieve sum rates that are close to the optimal algorithm, and higher than other suboptimal algorithms. Moreover, they have a low update frequency by purely utilizing statistical CSI, which contributes to a great reduction of computational burden.

The rest of this paper proceeds as follows. The MU-MIMO uplink system model is described in Section II. Section III presents the proposed metrics and the user selection algorithms. Simulation results are provided in Section IV and conclusions are drawn in Section V.

# II. SYSTEM MODEL

Consider a single-cell uplink MU-MIMO system. The BS is equipped with N antennas, and serves K users simultaneously.

Each user has M antennas. The BS is surrounded by  $N_s$  clusters, each of which may consist of several scatterers. The uplink signal transmitted by each user arrives at the BS via part or all of the clusters. Accordingly, the uplink flat fading channel of user  $k \in \{1, ..., K\}$  can be written as

$$\mathbf{G}_{k} = \sum_{n=1}^{N_{s}} \sqrt{c_{kn}} \mathbf{H}_{kn} \in \mathbb{C}^{N \times M}, \qquad (1)$$

where  $\mathbf{H}_{kn}$  denotes the subchannel from user k to the BS via cluster n, and it is normalized as  $\mathbb{E}\left[\|\mathbf{H}_{kn}\|_{F}^{2}\right] = 1$ .  $c_{kn}$  is a measure called *the significance of cluster n for user k* as in [8]. It measures the percentage of energy carried by cluster n in the channel of user k, and satisfies the following constraints:

$$0 \le c_{kn} \le 1 \left( k = 1, ..., K, n = 1, ..., N_s \right),$$
 (2a)

$$\sum_{n=1}^{N_s} c_{kn} = 1 \left( k = 1, ..., K \right).$$
 (2b)

Consequently, we have  $\mathbb{E}\left[\left\|\mathbf{G}_k\right\|_F^2\right] = 1.$ 

Regarding subchannel  $\mathbf{H}_{kn}$ , the physical MIMO channel model is considered. Assume cluster n consists of  $p_n$  discrete isotropic point scatterers, each of which corresponds to a path that arrives at BS. The path from user k to the BS via scatterer  $i \in \{1, ..., p_n\}$  of cluster n is characterized by a complex amplitude  $a_i^{(kn)}$ , a direction of departure (DOD)  $\beta_i^{(kn)}$ , and a DOA  $\alpha_i^{(kn)}$ . It is further assumed that  $\alpha_i^{(kn)}$  is invariant with user index k, thus we have  $\alpha_i^{(kn)} = \alpha_i^{(n)}$ . Accordingly,  $\mathbf{H}_{kn}$  can be written as

$$\mathbf{H}_{kn} = \sum_{i=1}^{p_n} a_i^{(kn)} \mathbf{e}_r \left( \alpha_i^{(n)} \right) \mathbf{e}_t^H \left( \beta_i^{(kn)} \right), \tag{3}$$

where  $a_i^{(kn)}$  follows independent Gaussian distribution with zero mean and a variance of  $1/p_n$ .  $\mathbf{e}_r(\alpha) \in \mathbb{C}^{N \times 1}$  and  $\mathbf{e}_t(\beta) \in \mathbb{C}^{M \times 1}$  are normalized steering vectors associated with DOA  $\alpha$  and DOD  $\beta$ , respectively [9].

Given the transmit power constraint P for each user and the noise power spectral density  $\sigma^2$  at receive antennas, we define the signal to noise ratio (SNR) as  $\gamma = P/M\sigma^2$ . Then based on the model above, the ergodic sum rate when applying dirty paper coding (DPC) is

$$R = \mathbb{E}\left[\log_2 \det\left(\mathbf{I}_N + \gamma \sum_{k=1}^K \mathbf{G}_k \mathbf{G}_k^H\right)\right].$$
 (4)

The closed form of the above ergodic sum rate is difficult to obtain. Consequently, the design of our user scheduling strategy is based on the capacity upper bound which can be obtained according to the concavity of  $\log_2 (\det (\mathbf{X}))$  function. The upper bound can be written as

$$R_{up} = \log_2 \det \left( \mathbf{I}_N + \gamma \sum_{k=1}^K \mathbb{E} \left[ \mathbf{G}_k \mathbf{G}_k^H \right] \right), \qquad (5)$$

where  $\mathbf{\Phi}_k = \mathbb{E}\left[\mathbf{G}_k\mathbf{G}_k^H\right]$  is in fact the spatial correlation matrix of channel  $\mathbf{G}_k$  at BS. According to (1) and (3), it can be written as

$$\mathbf{\Phi}_{k} = \sum_{n=1}^{N_{s}} \frac{c_{kn}}{p_{n}} \sum_{i=1}^{p_{n}} \mathbf{e}_{r} \left( \alpha_{i}^{(n)} \right) \mathbf{e}_{r}^{H} \left( \alpha_{i}^{(n)} \right).$$
(6)

Let  $\mathbf{R}_n = \frac{1}{p_n} \sum_{i=1}^{p_n} \mathbf{e}_r \left( \alpha_i^{(n)} \right) \mathbf{e}_r^H \left( \alpha_i^{(n)} \right)$ , then  $\mathbf{R}_n$  is the spatial correlation matrix of subchannel  $\mathbf{H}_{kn}$  at BS, and is invariant with index k. Hence we have

$$R_{up} = \log_2 \det \left( \mathbf{I}_N + \gamma \sum_{n=1}^{N_s} \mathbf{R}_n \sum_{k=1}^{K} c_{kn} \right).$$
(7)

Obviously, the upper bound  $R_{up}$  is a function of the significance coefficient  $c_{kn}$  and the spatial correlation matrix  $\mathbf{R}_n$ .

# III. LOW COMPLEXITY SCHEDULING DESIGN

In this section, two statistical CSI based user scheduling metrics are proposed. On one hand, users who transmit signals via more clusters are preferable, which contributes to lower intra-link spatial correlations. On the other hand, however, users sharing too many clusters causes an increase of inter-link correlations, which will reduce the sum rate. Consequently, a tradeoff between these two aspects is achieved by the proposed two metrics. Two user scheduling algorithms are further designed.

## A. Channel Parameter Estimation

The estimations of  $c_{kn}$  and  $\mathbf{R}_n$  will be briefly discussed in this subsection which serves as a foundation for the later metric design in subsection B. In order to estimate these parameters, pilot-aided uplink channel estimation is applied.

Consider  $T \in \mathbb{Z}^+$  consecutive or nonconsecutive time slots, during which  $c_{kn}$  and DOAs remain constant, but  $a_i^{(kn)}$  varies from slot to slot. Thus the signal received at BS in the *t*th  $(t \in \{1, ..., T\})$  time slot is

$$\mathbf{Y}(t) = \sum_{k=1}^{K} \mathbf{G}_{k}(t) \mathbf{X}_{k}(t) + \mathbf{N}(t), \qquad (8)$$

where  $\mathbf{X}_k(t) \in \mathbb{C}^{M \times MK}$  represents the pilot sequences transmitted by user k and  $\mathbf{N}(t)$  is the additive white Gaussian noise (AWGN) at receiver antennas. Orthogonality of  $\mathbf{X}_k$  requires  $\mathbf{X}_k(t) \mathbf{X}_k(t)^H = \mathbf{I}_{MK}$  and  $\mathbf{X}_k(t) \mathbf{X}_l(t)^H = \mathbf{0} \ (k \neq l)$ . Consequently,  $\mathbf{G}_k(t)$  can be estimated as

$$\hat{\mathbf{G}}_{k}(t) = \mathbf{Y}(t) \mathbf{X}_{k}^{H}(t) = \mathbf{G}_{k}(t) + \mathbf{N}(t) \mathbf{X}_{k}^{H}(t).$$
(9)

Let  $\eta_i^{(kn)}(t) = \sqrt{c_{kn}} a_i^{(kn)}(t)$ , then applying SAGE (spacealternating generalized expectation-maximization) algorithm to  $\hat{\mathbf{G}}_k(t)$  provides the estimators of  $\eta_i^{(kn)}(t)$ ,  $\alpha_i^{(n)}$  and  $\beta_i^{(kn)}$ , denoted as  $\hat{\eta}_i^{(kn)}(t)$ ,  $\hat{\alpha}_i^{(n)}(t)$  and  $\hat{\beta}_i^{(kn)}(t)$ , respectively [10].  $\hat{\alpha}_i^{(n)}(t)$  is tagged with time slot index t since the estimator can be inaccurate and vary from slot to slot. Further, virtual cluster (VC) is defined to tag each path with a proper cluster index. Specifically, when paths have DOA estimators that fall into the same angle range  $[\theta_{nl}, \theta_{nh}]$ , we consider them from VC n, even though they may be scattered from several different physical clusters in reality. Let  $S_n(t)$  be the set of paths that tagged with VC n in slot t,  $\mathbf{R}_n$  can be estimated as

$$\hat{\mathbf{R}}_{n} = \frac{1}{T} \sum_{t=1}^{T} \hat{\mathbf{H}}_{kn}(t) \,\hat{\mathbf{H}}_{kn}^{H}(t), \qquad (10)$$

where  $\hat{\mathbf{H}}_{kn}(t)$  can be obtained as

$$\hat{\mathbf{H}}_{kn}\left(t\right) = \sum_{i \in S_{n}\left(t\right)} \hat{a}_{i}^{\left(kn\right)}\left(t\right) \mathbf{e}_{r}\left(\hat{\alpha}_{i}^{\left(n\right)}\left(t\right)\right) \mathbf{e}_{t}^{H}\left(\hat{\beta}_{i}^{\left(kn\right)}\left(t\right)\right).$$
(11)

Based on Eq. (1)-(3),  $c_{kn}$  is related to  $\eta_i^{(kn)}$  as

$$c_{kn} = \mathbb{E}\left[\sum_{i \in \mathcal{S}_n} \left|\eta_i^{(kn)}\right|^2\right].$$
 (12)

As a result,  $c_{kn}$  can be estimated as

$$\hat{c}_{kn} = \frac{1}{T} \sum_{t=1}^{T} \sum_{i \in S_n(t)} \left| \hat{\eta}_i^{(kn)}(t) \right|^2.$$
(13)

#### B. User Selection Metric

Different from existing strategies which utilize instantaneous CSI, our metric emphasizes long term channel spatial structure, such as the spatial correlation matrix of subchannel via each VC and how users make use of these VCs to transmit signals. To find the best way to utilize the VCs, an optimization problem is first introduced below:

$$\mathcal{P}_{1}: \max_{c_{kn}} \hat{R}_{up} = \log_{2} \det \left( \mathbf{I}_{N} + \gamma \sum_{n=1}^{N_{s}} \hat{\mathbf{R}}_{n} \sum_{k=1}^{K_{0}} c_{kn} \right)$$
  
s.t.  $0 \le c_{kn} \le 1 \ (k = 1, ..., K_{0}, n = 1, ..., N_{s})$   
 $\sum_{n=1}^{N_{s}} c_{kn} = 1 \ (k = 1, ..., K_{0}),$ 

where  $K_0 \leq K$  is the number of users to be selected.

It can be proved that  $\mathcal{P}_1$  is a convex optimization problem by second-order condition, and thus its optimal solution can be calculated numerically with interior-point methods [11]. The detailed proof is omitted here due to the lack of space. Denoting the optimal solution as  $\{c_{kn}^*\}$ , it represents the best way for the  $K_0$  users to utilize the clusters in terms of maximizing sum rate upper bound. Consequently, the users whose  $c_{kn}$  are closer to  $c_{kn}^*$  should be selected with a higher probability. Define  $\mathbf{c}_k^* = [\mathbf{c}_{k1}^*, ..., \mathbf{c}_{kN_s}^*]$  as the optimal vector for the  $k^{th}$  user, and  $\mathbf{c}^* = [\mathbf{c}_1^*, ..., \mathbf{c}_{K_0}^*]$ . The closeness between a user subset and the optimal solution  $\mathbf{c}^*$  is measured by a distance to be introduced in the following.

Let  $x = C_K^m \ (m \le K_0)$  be the number of combinations of choosing m elements from  $\{1, ..., K\}$ , and  $\mathcal{K} = \{\mathcal{K}_1, ..., \mathcal{K}_x\}$  contains all possible combinations.  $\mathcal{K}_i \ (i \in \{1, ..., x\})$  is a set of m elements and represents the *i*th combination. We also let  $y = C_{K_0}^m$  be the number of combinations of choosing m elements from  $\{1, ..., K_0\}$ , and define  $\tilde{K} = \left\{\tilde{K}_1, ..., \tilde{K}_y\right\}$  and  $\tilde{K}_j \ (j = 1, ..., y)$  in a similar way. Let z = m! and

 $\mathcal{B}_i = {\mathbf{b}_{i1}, ..., \mathbf{b}_{iz}}$  denote all permutations of  $\mathcal{K}_i$ , with the vector  $\mathbf{b}_{ij} \in C^{1 \times m}$  being the *j*th permutation. Therefore, the distance between subset  $\mathcal{K}_i$  and  $\mathbf{c}^*$  is defined as

$$d\left(\mathcal{K}_{i}, \mathbf{c}^{*}; m\right) = \min_{\tilde{\mathcal{K}}_{i} \in \tilde{\mathcal{K}}} \min_{\mathbf{b} \in \mathcal{B}_{i}} \left( \sum_{j=1}^{m} \sum_{n=1}^{N_{s}} \left| c_{\tilde{\mathcal{K}}_{i}(j)n}^{*} - c_{b_{j}n} \right|^{p} \right)^{\frac{1}{p}},$$

$$(14)$$

where  $p \ge 1$  is a real number,  $\hat{\mathcal{K}}_i(j)$  is the *j*th element of  $\hat{\mathcal{K}}_i$ and  $b_j$  is the *j*th element of **b**.  $c_{b_jn}$  will be replaced by  $\hat{c}_{b_jn}$ when only its estimator is available.

Since having a smaller distance means the selected  $K_0$  users in  $\mathcal{K}_i$  use clusters in a better way, it seems that the smaller  $d(\mathcal{K}_i, \mathbf{c}^*; K_0)$  is, the better the subset  $\mathcal{K}_i$  will be. However, distance is not the only factor. The visibility of clusters to users is not embodied in  $\mathcal{P}_1$ . Consequently,  $\mathbf{c}^*$  is obtained on the basis of an assumption that all clusters are visible to all users. Hence, it is possible that  $c^*$  has very few zero elements when  $N_s$  is large. The reason is that transmitting signals via more clusters results in a larger DOA angle spread and more spatial degrees of freedom (DOFs), the benefit from which may outweigh the harm from sharing all clusters (which increases inter-link correlation and compromises sum rate). In practice, however, only a finite number of clusters are visible to a specific user based on the user location, which will greatly reduce the DOFs [12]. In this case, the increase of inter-link correlation resulted from sharing clusters cannot be neglected. In this paper, the inter-link correlation is evaluated by calculating the correlation matrix collinearity (CMC) as [13]

$$h'(k,l) = \frac{\left|\operatorname{tr}\left(\boldsymbol{\Phi}_{k}\boldsymbol{\Phi}_{l}^{H}\right)\right|}{\|\boldsymbol{\Phi}_{k}\|_{F}\|\boldsymbol{\Phi}_{l}\|_{F}} \in [0,1],$$
(15)

where  $\Phi_k$  is the intra-link spatial correlation matrix defined in (6), and tr (X) calculates the trace of matrix X. h'(k, l)describes how similar the subspaces of correlation matrices of user k and user l are. A smaller h'(k, l) indicates a lower inter-link correlation between user k and user j. The CMC of subset  $\mathcal{K}_i$  is defined as

$$h\left(\mathcal{K}_{i}\right) = \sum_{k,l \in \mathcal{K}_{i}, k \neq l} h'\left(k,l\right).$$
(16)

Based on the definitions as in (14) and (15), we propose the following metric for subset  $\mathcal{K}_i$  as

$$f(\mathcal{K}_i; m) = \alpha d(\mathcal{K}_i, \mathbf{c}^*; m) + (1 - \alpha) h(\mathcal{K}_i), \qquad (17)$$

where  $\alpha \in [0, 1]$  is a weighting coefficient.

The target is to find the optimal  $K_0$  users so as to minimize f as follows

$$\mathcal{P}_2: \quad \mathcal{K}_i^* = \arg\min f\left(\mathcal{K}_i; K_0\right)$$
  
s.t.  $\mathcal{K}_i \in \mathcal{K}.$ 

## C. Cluster Information based Algorithms

The optimal solution of  $\mathcal{P}_2$  can be obtained with a bruteforce approach to search over all subsets in  $\mathcal{K}$ . However, the complexity burden is prohibitive when K and  $K_0$  are large. Therefore, a suboptimal low complexity selection algorithm which iteratively selects users is proposed. Assume that the optimal significance vector c\* has been obtained before user selection, and it is regarded as one of the inputs. The detailed process is described as follows.

# Algorithm 1 : Cluster Information based Algorithm Input:

The estimators of cluster correlation matrices:  $\ddot{\mathbf{R}}_n$ ; The estimators of significance coefficients:  $\hat{c}_{kn}$ ; The optimal significance vector  $c^*$ ; User number K and selected user number  $K_0$ ; Metric parameters: p and  $\alpha$ ; Step 1:

- 1: Initialize:  $\Omega = \{1, ..., K\}, \Upsilon = \emptyset$ , and m = 1.
- 2: Obtain set  $\tilde{\mathcal{K}}$  as defined.
- 3: Let  $s_1 = \arg\min_{i \in \Omega} d(\{i\}, \mathbf{c}^*; m)$ . 4: Update:  $\Omega \leftarrow \Omega \{s_1\}$  and  $\Upsilon \leftarrow \Upsilon + \{s_1\}$ .
- 5: Calculate the capacity upper bound when serving user subset  $\Upsilon$ ,  $R_{up}(\Upsilon)$ .

#### Step 2:

- 6: for  $m = 2: K_0$  do
- 7: Update set  $\mathcal{K}$ .
- 8:
- $$\begin{split} \hat{s_m} &= \arg\min_{i\in\Omega} f\left(\Upsilon \cup \{i\}; m\right).\\ \text{Calculate the capacity upper bound } R_{up}\left(\Upsilon \cup \{s_m\}\right). \end{split}$$
  9:
- 10: if  $R_{up}\left(\Upsilon \cup \{s_m\}\right) \leq R_{up}\left(\Upsilon\right)$  then
- go to step 3; 11: 12:

 $R_{up}\left(\Upsilon\right)\leftarrow R_{up}\left(\Upsilon\cup\left\{s_{m}
ight\}
ight).$ end if 13:

- 14:
- Update:  $\Omega \leftarrow \Omega \{s_m\}$  and  $\Upsilon \leftarrow \Upsilon + \{s_m\}$ . 15:

16: end for

Step 3:

## 17: Terminate the algorithm. **Output:**

The selected user subset:  $\Upsilon$ .

It can be seen that the algorithm avoids time-consuming SVD operations. In addition, it is only based on statistical CSI. Thus the selected user subset will not be updated until the statistical CSI changes, which contributes to a great computational burden reduction.

However, the distance in (14) involves comparisons over all permutations of  $\mathcal{K}_i$  and all combinations  $\tilde{K}_i$ , which leads to a large searching space when  $K_0$  and K are large. Moreover, the distance does not support recursive calculation from the previous iterations. As a result, the search has to be executed in each iteration. To further reduce computational complexity, a distance that supports recursive calculation is proposed.

Denote  $d_1(\{i\}, \mathbf{c}_i^*)$  as the distance between user *i* and vector  $\mathbf{c}_{i}^{*}$ , which can be expressed as follows

$$d_1\left(\{i\}, \mathbf{c}_j^*\right) = \left(\sum_{n=1}^{N_s} \left|c_{jn}^* - c_{in}\right|^p\right)^{\frac{1}{p}}.$$
 (18)

Let  $\Omega_0 = \{1, ..., K_0\}$  and  $\Psi \subseteq \Omega_0$ , then the distance between user *i* and the optimal vector  $\mathbf{c}^*$  on the support of  $\Psi$  is defined as

$$d_{2}(\{i\}, \mathbf{c}^{*}; \Psi) = \min_{j \in \Psi} d_{1}(\{i\}, \mathbf{c}_{j}^{*}).$$
(19)

Let  $l_i = \arg\min_{j \in \Psi} d_1(\{i\}, \mathbf{c}_j^*)$ , then the distance between subset  $\mathcal{K}_i$  and  $\mathbf{c}^*$  is defined as

$$\tilde{d}\left(\mathcal{K}_{i}, \mathbf{c}^{*}; m\right) = \left(\sum_{k=1}^{m} d_{2}\left(\left\{\mathcal{K}_{i}\left(k\right)\right\}, \mathbf{c}^{*}; \Omega_{0} - \Upsilon_{k-1}^{\left(i\right)}\right)^{p}\right)^{\frac{1}{p}},$$
(20)

where  $\Upsilon_k^{(i)} = \bigcup_{j=1}^k \{ l_{\mathcal{K}_i(j)} \}$  and  $\Upsilon_0^{(i)} = \emptyset$ .  $\Omega_0 - \Upsilon_{k-1}^{(i)}$  means removing the elements of  $\Upsilon_{k-1}^{(i)}$  from  $\Omega_0$ .

Based on the definition as in (20), it is straightforward to see that distance  $\tilde{d}$  supports recursive calculation as

$$\tilde{d}(K_{i} + \{j\}, \mathbf{c}^{*}; m+1) = \left(\tilde{d}(K_{i}, \mathbf{c}^{*}; m)^{p} + d_{2}\left(\{j\}, \mathbf{c}^{*}; \Omega_{0} - \Upsilon_{m-1}^{(i)} \cup \{l_{K_{i}(m)}\}\right)^{p}\right)^{\frac{1}{p}},$$
(21)

where  $\mathcal{K}_i + \{j\}$  means adding j to  $\mathcal{K}_i$  as its last element. Based on this distance definition, we propose a less tight metric as

$$\tilde{f}(\mathcal{K}_{i};m) = \alpha \tilde{d}(\mathcal{K}_{i},\mathbf{c}^{*};m) + (1-\alpha)h(\mathcal{K}_{i}).$$
(22)

The target is to select the best  $K_0$  users so as to minimize f as  $\mathcal{P}_3$ . An algorithm is further proposed in next page.

$$\mathcal{P}_3: \quad \mathcal{K}_i^* = \arg\min f\left(\mathcal{K}_i; K_0\right)$$
  
s.t.  $\mathcal{K}_i \in \mathcal{K}.$ 

As shown in Line 7 of Algorithm 2, distance  $d_{tp}$  of the current iteration can be calculated from that of the previous iteration and a simple distance  $d_2$ , which greatly narrows down the searching space. From this point, we can see that Algorithm 2 is of great lower complexity than Algorithm 1.

#### D. Computational Complexity Analysis

We evaluate the computational complexity of the proposed two algorithms in terms of the number of flops. A real addition, multiplication, or division is counted as one flop. p = 1 is considered here as an example to shed some light on the complexity comparison. As shown in algorithms, the complexity mainly comes from the calculation of distance d or d, CMC h and the iterations. The detailed analysis is omitted here, and only results are presented. After calculating step by step, we obtain that the complexity lower bound of Algorithm 1 is  $\mathcal{O}\left(N_s K_0^{(K_0+1)} + N^2 K_0^4\right)$ ; the complexity upper bound and lower bound of Algorithm 2 are  $\mathcal{O}\left(\left(N_s + N^2\right)K^3\right)$ ,  $\mathcal{O}((N_s + N^2) K_0^3)$ , respectively. It can be seen that both two algorithms have quadratic complexity with BS antenna number N, instead of  $N^3$  in SVD-involved algorithms, which brings in a great complexity reduction when N is large. In addition, the complexity upper bound of Algorithm 2 increases cubically as user number K grows large, while the lower bound of Algorithm 1 increases exponentially as

Algorithm 2 : Simplified Cluster Information based Algorithm

 $\hat{\mathbf{R}}_n, \hat{c}_{kn}, \mathbf{c}^*, K_0, p \text{ and } \alpha.$ Input: Step 1: 1: Initialize:  $\Omega = \{1, ..., K\}, \Omega_0 = \{1, ..., K_0\};$  $\Upsilon = \emptyset, \ \Upsilon_0 = \emptyset \text{ and } m = 1.$  $1 - \psi, \ 10 - \psi \text{ and } m = 1.$ 2: Let  $s_1 = \arg\min_{i \in \Omega} d_2(\{i\}, \mathbf{c}^*; \Omega_0).$ 3: Let  $l_1 = \arg\min_{j \in \Omega_0} d_1(\{s_1\}, \mathbf{c}_j^*), \ d_{tp} = d_1(\{s_1\}, \mathbf{c}_{l_1}^*).$ 4: Update:  $\Omega \leftarrow \Omega - \{s_1\}, \ \Upsilon \leftarrow \Upsilon + \{s_1\};$   $\Omega_0 \leftarrow \Omega_0 - \{l_1\}, \ \Upsilon_0 \leftarrow \Upsilon_0 + \{l_1\}.$ 5: Calculate  $R_{up}(\Upsilon)$  and  $h(\Upsilon).$ Step 2: 6: for  $m = 2: K_0$  do 7:  $s_m = \arg\min_{i\in\Omega} \left\{ \alpha [d_{tp}{}^p + d_2(\{i\}, \mathbf{c}^*; \Omega_0)^p]^{1/p} + (1 - \alpha) [h(\Upsilon) + \sum_{k\in\Upsilon} h'(k, i)] \right\}.$ 8: Calculate upper bound  $R_{up}(\Upsilon \cup \{s_m\})$ , and  $h(\Upsilon)$ . if  $R_{up}(\Upsilon \cup \{s_m\}) \leq R_{up}(\Upsilon)$  then 9: go to step 3; 10: 11: else  $R_{up}\left(\Upsilon\right)\leftarrow R_{up}\left(\Upsilon\cup\left\{s_{m}
ight\}
ight).$ end if 12: 13: Let  $l_m = \arg\min_{j\in\Omega_0} d_1\left(\{s_m\}, \mathbf{c}_j^*\right).$ 14: Update:  $d_{tp} \leftarrow \left[d_{tp}{}^{p} + d_{1}\left(\{s_{m}\}, \mathbf{c}_{l_{m}}^{*}\right)^{p}\right]^{\frac{1}{p}}$ . Update:  $\Omega \leftarrow \Omega - \{s_{m}\}, \Upsilon \leftarrow \Upsilon + \{s_{m}\};$  $\Omega_{0} \leftarrow \Omega_{0} - \{l_{m}\}, \text{ and } \Upsilon_{0} \leftarrow \Upsilon_{0} + \{l_{m}\}.$ 15: 16: 17: end for Step 3: 18: Terminate the algorithm. **Output:** The selected user subset:  $\Upsilon$ .

 $K_0$  grows. Consequently, Algorithm 2 is of significant lower computational complexity than Algorithm 1, when K and  $K_0$  are very large and have the same order of magnitude.

#### **IV. SIMULATION RESULTS**

In this section, numerical results are provided to evaluate the proposed algorithms. Perfect channel estimation is assumed.

The simulation scenario is firstly presented as in Fig. 1. The units of both axes are in meters. A linear array is located at the BS, vertical to x-axis. Thus we only consider the left half of the cell, the radius r of which is 500 meters. The locations of  $N_s$  clusters and K users are generated randomly via uniform distribution. The cluster-to-BS distance is  $50 \le d_s \le 200$ , and the user-to-BS distance is  $200 \le d_u \le 500$ . The radius of a cluster is 10 meters. Each cluster provides 20 paths.

In real propagation environment, the visibility of a cluster to a user is based on their locations. In our simulation, the visibility is manually set. For example, four users in Fig. 1 are set to have access to one cluster which is nearest to them (the dashed lines for user 1-4), and three users have access to two nearest clusters (the dotted lines for user 5-7). For user



Figure 1. Simulation scenario for K = 7,  $N_s = 4$ . A dotted or dashed line between a cluster and a user indicates the cluster is visible to the user.



Figure 2. Sum rates with K = 7,  $K_0 = 4$ ,  $N_s = 4$ , M = 8 and N = 32.

5-7, the significance coefficients of the two visible clusters are 0.2 and 0.8, with no specific order.

In this simulation, ergodic sum rates of the proposed algorithms are evaluated. They are obtained by averaging over 1000 independent location generations, in each of which 1000 independent flat fading channel realizations are generated. Performances of brute force algorithm (brute force alg), capacity based algorithm (capacity base alg), and chordal distance based algorithm (chordal distance alg) are also provided for comparison. Given p = 1 and different values of  $\alpha$ , simulation results for N = 32, M = 8, K = 7,  $K_0 = 4$ , and  $N_s = 4$  are shown in Fig. 2. Several conclusions can be drawn from it.

• Firstly, significant performance gains are achieved from the proposed two algorithms. For each value of  $\alpha$ , their sum rates are higher than chordal distance based algorithm. With  $\alpha = 0.7$  and SNR being 15dB, they achieve sum rates almost the same as that of brute force algorithm, 1.5 bit/s/Hz and 6 bit/s/Hz higher than those

of capacity based algorithm and chordal distance based algorithm, respectively.

- Secondly, the performances of the proposed two algorithms vary with the value of  $\alpha$ . In this simulation,  $\alpha = 0.7$  brings the highest sum rates for the two algorithms. The observation that  $\alpha = 1$  does not bring the highest rates justifies our consideration about interlink correlation.
- Furthermore, although based on a less tight metric, the optimal performance of Algorithm 2 can be very close to that of Algorithm 1. When α = 0.7, capacity of Algorithm 2 is only 0.5 bit/s/Hz smaller than Algorithm 1. More importantly, Algorithm 2 has a much lower complexity, thus it is of a much higher practical value.

Increasing user number and cluster number to 10 and 5, respectively, we present the results as in Fig. 3. Five users have access to one nearest cluster, and the other five to nearest two. We choose five users out of ten.



Figure 3. Sum rates with K = 10,  $K_0 = 5$ ,  $N_s = 5$ , M = 8 and N = 40.

The conclusions drawn from Fig. 3 are similar to those from Fig. 2, except that  $\alpha = 1$  instead of  $\alpha = 0.7$  brings the highest sum rates in the proposed algorithms. One possible reason is that with more clusters distributed around BS, using more clusters increases DOFs significantly, the benefit from which outweighs the harm from higher inter-link correlation. Therefore, it is necessary to assign a proper value to  $\alpha$  according to the specific scenario. The optimization algorithm of  $\alpha$  will be considered in the future work.

### V. CONCLUSIONS

In this paper, two user scheduling metrics for MU-MIMO systems in a finite scattering environment are proposed. By making use of long term spatial structure of user channels, the proposed metrics can achieve a tradeoff between decreasing intra-link correlation and increasing inter-link correlation. Furthermore, two user selection algorithms which select users through iterations are designed. Simulation results show that our algorithms achieve higher sum rates than chordal-distancebased algorithm and capacity algorithm, and the performances can be close to that of brute force algorithm with a proper parameter value. Last but not the least, our algorithms are of high practical value since they are of low computational complexity, especially the second algorithm, and the selected user subsets can be updated in a low frequency.

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